CreditGrades™
TECHNICAL DOCUMENT

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Foreword

This document is the technical discussion of the CreditGrades™ model for quantitative credit assessment. The model is a straightforward, practical application of the structural model for credit risk that has been used for a number of years by a variety of credit market participants. In designing the model, the authors of this document have made assumptions so as to relate relevant model parameters to market observables. It is the hope of the four institutions involved in the project — Deutsche Bank, Goldman Sachs, JPMorgan and RiskMetrics Group — that by documenting the details of the model here and by providing access to model outputs at `www.creditgrades.com`, we create a standard of transparency in the credit markets.

We would like to recognize Jean-Pierre Lardy of JPMorgan for providing the initial vision for this project and thank Jorge Mina of RiskMetrics Group for a careful review of the manuscript.

We encourage our readers to provide feedback or submit questions to `techdoc@creditgrades.com`

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Chapter 1

Introduction

In 1997, JPMorgan, with the co-sponsorship of five other institutions, published *CreditMetrics®*, a model for the calculation of portfolio credit risk. Among the main goals of the publication was to create an open, transparent standard and demonstrate to the industry (and not least to regulators) that market participants were able to manage credit risk at the portfolio level. Five years later, the model stands as the standard, and its acceptance has played a significant role in the discussion of the next generation in regulatory capital standards.

In 2002, there is a further need for open standards in the credit markets, but specifically for the quantitative assessment of single-name credit risk. The regulatory dialogue continues to be an impetus for such standards, but now, more so than in 1997, it is the markets themselves that are demanding this development. Since 1997, the expansion of the credit markets has continued. At the same time, new avenues for taking credit risk have emerged, from straightforward credit derivative products (where year-on-year growth in single name Credit Default Swaps has exceeded 50 percent over the last five years) to complex structured products (where issuance of Collateralized Debt Obligations continues to set records). This expansion of the markets has prompted the entry of market participants beyond the traditional banking institutions, and now includes derivatives players, asset managers, hedge funds, and insurance and reinsurance institutions.

Meanwhile, 2001 represents the most costly year ever in terms of corporate defaults. Broad market indicators were bad, with average default rates higher than at any time in a decade. Moreover, as Moody's notes[1], "Defaults in 2001 were notable for their individual size as well as their frequency. Twenty-nine issuers had defaults totaling over one billion dollars in debt apiece." Topping this off, of course, was Enron, with the largest bankruptcy filing on record.

The goal of CreditGrades™, then, is to address an expanding market replete with opportunities but also with risks. Our aim is to provide a transparent standard for quantitative credit risk. The CreditGrades model is

[1] See Hamilton, Cantor and Ou (2002).

an implementation of a set of techniques that have been in use within a number of broker-dealers. In this implementation, we have strived for accuracy as well as for a standardization and consistency of approach that allows the immediate application of the model to a wide array of publicly traded firms. Applications of the model include price discovery for illiquid firms, monitoring across a large array of firms, and the investigation of relative value opportunities across the credit and equity markets.

While the goals of accuracy and transparency are at times at odds, we believe that the CreditGrades model can be broadly accurate and at the same time understandable by a wide variety of market participants. We address three notions of model accuracy in this document: first, we choose our parameter estimation techniques to achieve the best possible tracking of market spreads; second, we test that the model performs well as a ranking tool in differentiating between high and low credit risks; finally, we examine a number of examples that track the model against information provided by the rating agencies and news releases regarding the health of the firms in question.

To achieve transparency, clearly it is important that we describe the mathematics of the model and the techniques used to estimate model parameters. Further, and just as importantly, we believe that it is crucial that when the model is not accurate that there be a set of well understood parameters that users can examine for the source of the inaccuracy. To this end, the assumptions made in the model have been designed to be accurate as a predictor of market levels, but also to produce a model where the inputs are purely market observables and where the impact of changes in the input variables are clear. Additionally, by exploring the model from each of these angles, we establish a strong notion of the level of accuracy to be expected. And by presenting two examples of cases where the model diverged from the actual market spreads, we illustrate how a user of the model might consider adjustments to the model parameters to reflect current market conditions.

Though it is hard to pinpoint a beginning to the modern history of quantitative credit assessment, a serious candidate is Altman (1968). In this approach, a number of firms are identified, of which some defaulted and some did not. The goal of the analysis is then to examine the firms at a time before the defaults occurred and identify which descriptors of the firms might have best helped distinguish the defaulting firms from the ones that did not default.

A second class of models, referred to as reduced form models, uses information from actual credit prices to extract default probabilities. Jarrow and Turnbull (1995) and Duffie and Singleton (1999) are commonly cited examples. These models do not attempt to explain the "why" of default likelihood, but only how the market views individual credits. As such, reduced form models are useful to compare opportunities across different forms of credit risk — for example, bonds versus credit derivatives or individual firms versus structured products — but cannot provide a view contrary to the market or suggest a price where no market exists.

A third class of models — the one to which CreditGrades belongs — is the class of structural models. These models derive from work of Black and Scholes (1973) and Merton (1974) who observed that both equity and debt can be viewed as options on the value of a firm's assets, implying that equity option pricing techniques can be adapted for use in assessing credit. Structural models are attractive in that they utilize information from a very broad and liquid market. Thus, structural models afford an alternative view of credit quality for firms whose credit trades actively and a starting price indicator for firms whose credit is illiquid or does not trade at all.

A number of other commercial implementations of the structural model are available, from which CreditGrades differs in two respects. The first is the goal of the models and subsequently the data used to train the models. Other commercial models are designed to produce accurate probabilities of default and to distinguish firms about to default from healthy firms. As such, the models are trained on proprietary default databases. The CreditGrades model is designed to track credit spreads well and to provide a timely indication of when a firm's credit becomes impaired. Parameter estimates and other model decisions were made based on the model's ability to reproduce historical default swap spreads. Since the modeling aim (accurate spreads versus accurate default probabilities) and training data (market spreads versus actual defaults) differ, CreditGrades offers a view of credit quality that is complementary to that of other models.

A second distinction is in how the model input parameters are derived. The current approaches take literal interpretations of the structural model approach. Consequently, there is a significant emphasis on how to calculate certain fundamental but unobservable parameters, notably the value and volatility of a firm's assets. The CreditGrades approach is more practical, bypassing strict definitions in favor of simple formulas tied to market observables. As a result, the CreditGrades model can be stated as a simple formula based on a small number of input parameters, and sensitivities to these parameters can be easily ascertained.

In the chapters to follow, our goal is to describe the fundamental assumptions of the CreditGrades model and, just as importantly, to discuss the implementation and data issues that arise in putting the approach into practice. We describe the modeling foundation in Chapter 2. In Chapter 3, we discuss parameter estimation issues and test the model's performance against a number of alternative approaches. These two chapters are technical in nature and are intended to provide a full detail of how the model is implemented.

Finally, in Chapter 4, we examine CreditGrades from a practical perspective. Here, we discuss what level of accuracy can be reasonably expected and the sort of general applications to which the model is suited. We also take up six different firms as examples, examine the model's performance in each case, and in cases of model divergence discuss how a user might interpret the results and go about adjusting model parameters.

Chapter 2

Model Description

The purpose of the CreditGrades™ model is to establish a robust but simple framework linking the credit and equity markets. The relationship between corporate debt and equity was first formally proposed by Black and Scholes (1973) and Merton (1974). These authors observed that equity may be modeled as an option on a firm's assets, and that the value of a firm's debt is simply the value of its assets in excess of the equity value. The approach was further developed by Black and Cox (1976) and later by Leland (1994). According to their approach (which we will refer to as the *structural model*), an event of default occurs when the asset value of a firm crosses a predetermined default barrier or threshold.

We use the structural model framework to develop a link between credit and equity derivatives. For the most part, the CreditGrades model can be viewed as a practical implementation of the standard structural model. We employ approximations for the asset value, volatility and drift terms which relate all of these quantities to market observables. In this framework, we value credit as an exotic equity derivative whose pricing formula can be expressed in closed form. The resulting formula is appealingly simple and yet can approximate any sophisticated model relying on similar fundamental assumptions. See Finkelstein (2001), Finkelstein and Lardy (2001), Lardy (2001a), Lardy (2001b), Lardy and Pradier (2001) and Pan (2001) for further detail.

One departure from the standard structural model we make is to address its artificially low short-term spreads. These low spreads occur because assets that begin above the barrier cannot reach the barrier immediately by diffusion only. Hull and White (2001) confront this issue using a time-dependent default barrier which is calibrated to market spreads. An alternative approach is to incorporate jumps into the asset value process. In our approach, we model the uncertainty in the default barrier, motivated by the fact that we cannot expect to know the exact level of leverage of a firm except at the time the firm actually defaults. The uncertainty in the barrier admits the possibility that the firm's asset value may be closer to the default point than we might otherwise believe. This leads to higher short-term spreads than are produced without the barrier uncertainty. Thus the standard deviation of recovery value takes on an important role in the calculation of the probability of default and its term structure.

2.1 Model description

The basic assumptions of our model are illustrated in Figure 2.1. We suppose a stochastic process V and define default as the first time V crosses the default barrier. V may be thought of intuitively as the asset value (on a per share basis) process for the firm, although as we will discuss below, we will not identify V exactly with the firm's asset value. We define the default barrier as the amount of the firm's assets that remain in the case of default. This quantity is simply the recovery value that the debt holders receive, $L \cdot D$, where L is the average recovery on the debt and D is the firm's debt-per-share.

We assume that the asset value evolves as a geometric Brownian motion

$$\frac{dV_t}{V_t} = \sigma dW_t + \mu_D dt, \tag{2.1}$$

where W is a standard Brownian motion, σ is the asset volatility, and μ_D is the asset drift. We assume for now that $\mu_D = 0$; we justify this in Section 2.2.

Because the standard structural model, with the asset value evolving by pure diffusion and the default barrier fixed, produces unrealistic short-term credit spreads, we introduce randomness to the average recovery value L. The introduction of uncertain recovery value is based on empirical studies of recovery rates.[1] One prevalent finding of these studies is an extreme variance of the distribution of recoveries. In addition to some industrial sector dependence, the recovery rate can be greatly affected by factors such as whether default is triggered by financial or operational difficulties and whether the company will be restructured or liquidated.

We assume that the recovery rate L follows a lognormal distribution with mean $\bar{L}$ and percentage standard deviation λ. Specifically,

$$\begin{aligned} \bar{L} &= \mathbf{E}L, & (2.2)\\ \lambda^2 &= \operatorname{Var}\log(L), \text{ and} & (2.3)\\ LD &= \bar{L}De^{\lambda Z - \lambda^2/2}, & (2.4) \end{aligned}$$

where Z is a standard normal random variable.[2] The random variable Z is independent of the Brownian motion W. Z is unknown at $t = 0$ and is only revealed at the time of default.[3] Intuitively, by letting Z be random, we are capturing the uncertainty in the actual level of a firm's debt-per-share. Thus, there is some true level of L that does not evolve through time, but that we are unable to observe with certainty. With the uncertain recovery rate, the default barrier can be hit unexpectedly, resulting in a jump-like default event.

For an initial asset value V_0, default does not occur as long as

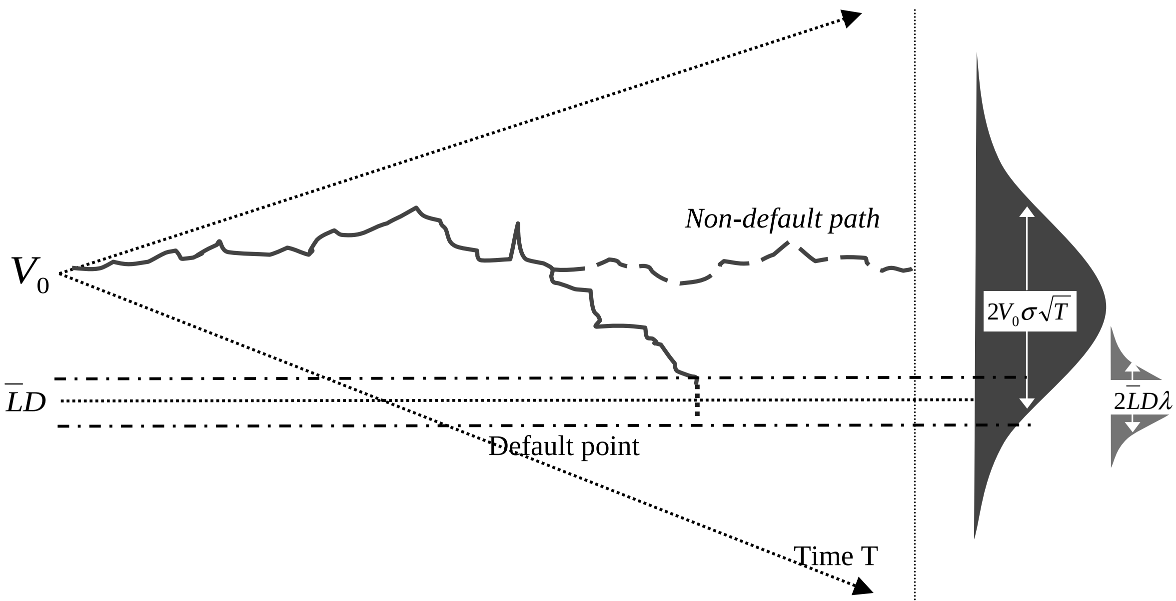
$$V_0 e^{\sigma W_t - \sigma^2 t/2} > \bar{L}De^{\lambda Z - \lambda^2/2}. \tag{2.5}$$

[1] For example, Hu and Lawrence (2000).

[2] We adopt the convention that log denotes the natural logarithm.

[3] Technically, there is a filtration F to which W is adapted such that Z is independent of F_0 but $Z \in \mathsf{F}_t$ for all $t > 0$.

Figure 2.1: Model description



The survival probability of the company at time t is then given by the probability that the asset value (2.1) does not reach the barrier (2.4) before time t.

Introducing a process

$$X_t = \sigma W_t - \lambda Z - \frac{\sigma^2 t}{2} - \frac{\lambda^2}{2}, \tag{2.6}$$

we rewrite (2.5) as

$$X_t > \log(\bar{L}D/V_0) - \lambda^2. \tag{2.7}$$

Notice that for $t \geq 0$, X_t is normally distributed with

$$\mathbf{E}X_t = -\frac{\sigma^2}{2}\left(t + \lambda^2/\sigma^2\right), \tag{2.8}$$

$$\operatorname{Var} X_t = \sigma^2\left(t + \lambda^2/\sigma^2\right). \tag{2.9}$$

Note that if $\lambda \neq 0$, X_0 has positive variance. We approximate the process X with a Brownian motion $\hat{X}$ with drift $-\sigma^2/2$ and variance rate σ^2. We stipulate that $\hat{X}$ starts in the past at $-\Delta t = -\lambda^2/\sigma^2$ with $\hat{X}_{-\Delta t} = 0$. It can be seen that for $t \geq 0$, the moments of $\hat{X}_t$ agree with the moments of X_t above. Intuitively, our approximation replaces the uncertainty in the default barrier with an uncertainty in the level of the asset value at time 0; since it is the distance between the asset value and the default barrier that ultimately drives the model, this approximation has little impact.

We now make use of the distributions for first hitting time of Brownian motion. In particular, for the process $Y_t = at + bW_t$ with constant a and b, we have (see, for example, Musiela and Rutkowski (1998))

$$\mathbf{P}\{Y_s > y, \forall s < t\} = \Phi\left(\frac{at - y}{b\sqrt{t}}\right) - e^{2ay/b^2}\Phi\left(\frac{at + y}{b\sqrt{t}}\right). \tag{2.10}$$

To apply this result to $\hat{X}$, we set $a = -\sigma^2/2$, $b = \sigma$ and $y = \log(\bar{L}D/V_0) - \lambda^2$, and substitute t with $t + \lambda^2/\sigma^2$, we obtain a closed form formula for the survival probability up to time t,

$$P(t) = \Phi\left(-\frac{A_t}{2} + \frac{\log(d)}{A_t}\right) - d \cdot \Phi\left(-\frac{A_t}{2} - \frac{\log(d)}{A_t}\right), \tag{2.11}$$

where

$$d = \frac{V_0 e^{\lambda^2}}{\bar{L}D}, \tag{2.12}$$

$$A_t^2 = \sigma^2 t + \lambda^2. \tag{2.13}$$

Note that the survival probability given by (2.11) implicitly includes the possibility of default in the period $(-\Delta t, 0]$, producing counterintuitive result that there is a non-zero probability of default at $t = 0$. This particular fact may be considered a technical artifact of the modeling assumptions, specifically the lognormality of the default barrier. At the same time, though, this feature aids in obtaining a simple formula for survival probability and in producing reasonable spreads for short (6-month to 2-year) maturity instruments.

An alternative to the approximation with $\hat{X}$ is to integrate over the barrier distribution. This approach yields an expression for the survival probability that contains the cumulative bivariate normal distribution:

$$P(t) = \Phi_2\left(-\frac{\lambda}{2} + \frac{\log(d)}{\lambda}, -\frac{A_t}{2} + \frac{\log(d)}{\lambda}; \frac{\lambda}{A_t}\right) - d \cdot \Phi_2\left(\frac{\lambda}{2} + \frac{\log(d)}{\lambda}, -\frac{A_t}{2} - \frac{\log(d)}{\lambda}; -\frac{\lambda}{A_t}\right). \quad (2.14)$$

For practical purposes, the numerical differences between the survival probabilities given by the two approaches are marginal.

To convert the CreditGrades survival probability to a credit price, we must specify two additional parameters: the riskfree interest rate r and the recovery rate R on the underlying credit. Note that R differs from $\bar{L}$ in that R is the expected recovery on a specific class of a firm's debt, while $\bar{L}$ is the expected recovery averaged over all debt classes. The asset specific recovery R for an unsecured debt is usually lower than $\bar{L}$ since the secured debt will have a higher recovery.

To price a Credit Default Swap (CDS), we solve for the continuously compounded spread c^* such that the expected premium payments on the CDS equate to the expected loss payouts. For a constant risk-free interest rate r and the survival probability function given by the CreditGrades model, the par spread for a CDS with maturity t may be expressed as

$$c^* = r(1-R)\frac{1 - P(0) + e^{r\xi}(G(t+\xi) - G(\xi))}{P(0) - P(t)e^{-rt} - e^{r\xi}(G(t+\xi) - G(\xi))}, \quad (2.15)$$

where $\xi = \lambda^2/\sigma^2$, and the function G is given by Rubinstein and Reiner (1991):

$$G(u) = d^{z+1/2}\Phi\left(-\frac{\log(d)}{\sigma\sqrt{u}} - z\sigma\sqrt{u}\right) + d^{-z+1/2}\Phi\left(-\frac{\log(d)}{\sigma\sqrt{u}} + z\sigma\sqrt{u}\right), \quad (2.16)$$

with $z = \sqrt{1/4 + 2r/\sigma^2}$. Details of the spread calculation are given in Appendix A.

In practice, we see little difference between spreads calculated by assuming continuous fee payments and those calculated using the market standard of quarterly payments. For simplicity, we calculate the CreditGrades spread as above and adjust for the market's Act/360 pricing convention.

2.2 Calibrating model parameters

In order to implement the survival probability formula (2.11), it is necessary to link the initial asset value V_0 and the asset volatility σ to market observables. We accomplish this by examining the boundary conditions. We focus on long-term tenors ($t > \lambda^2/\sigma^2$), since the short-term default probability is mainly driven by the level of λ.

Let S denote the firm's equity price and σ_S the equity volatility. In general, the equity and asset volatilities are related through

$$\sigma_S = \sigma \frac{V}{S} \frac{\partial S}{\partial V}. \tag{2.17}$$

Define the distance to default measure η as the number of annualized standard deviations separating the firm's current equity value from the default threshold:

$$\eta = \frac{1}{\sigma} \log\left(\frac{V}{LD}\right) = \frac{V}{\sigma_S S} \frac{\partial S}{\partial V} \log\left(\frac{V}{LD}\right). \tag{2.18}$$

Clearly, η plays an important role in determining the survival probability through (2.11), and so we will focus on the behavior of η in our boundary cases.

The first boundary condition is the behavior of V near the default threshold $L \cdot D$. We assume that as default approaches (that is, $S/(LD) \ll 1$) , the value of the equity (which we denote by S) approaches zero. Thus,

$$V|_{S=0} = LD, \tag{2.19}$$

at the boundary and

$$V \approx L \cdot D + \frac{\partial V}{\partial S} S, \tag{2.20}$$

near the default threshold. Substituting into (2.18), we see that

$$\eta \approx 1/\sigma_S, \tag{2.21}$$

near the boundary.

The second boundary condition is far from the default barrier (that is, $S \gg LD$). Here, we assume

$$S/V \to 1, \tag{2.22}$$

that is that the asset and equity values increase at the same rate. This leads to an approximation for η:

$$\eta \simeq \frac{1}{\sigma_S} \log\left(\frac{S}{LD}\right). \tag{2.23}$$

The simplest expressions for V and η that simultaneously satisfies the near default boundary conditions ((2.19) and (2.21)) and the far from default conditions ((2.22) and (2.23)) are $V = S + LD$ and

$$\eta = \frac{S + LD}{\sigma_S S} \log\left(\frac{S + LD}{LD}\right). \tag{2.24}$$

Thus, for the initial asset value V_0 at time $t = 0$, we have

$$V_0 = S_0 + \bar{L}D, \tag{2.25}$$

where S_0 is the current stock price. This also gives

$$\sigma = \sigma_S \frac{S}{S + \bar{L}D}, \tag{2.26}$$

relating the asset volatility to the observable equity volatility.

Equation (2.26) shows that for a stable asset volatility, the equity volatility increases with declining stock price, and eventually reaches very high levels for a company at the brink of default. This dependence of equity volatility on the stock price is evident in a pronounced volatility skew in equity option markets, especially for high yield names. It often makes sense to use a reference share price S^* and equity volatility σ_S^* (either historical or implied) to determine an asset volatility and keep it stable for some period of time. In this case, the asset volatility will be given by

$$\sigma = \sigma_S^* \frac{S^*}{S^* + \bar{L}D} \tag{2.27}$$

The estimation of σ is the subject of Section 3.1.

In deriving (2.11), another assumption has been that the asset value has zero drift ($\mu_V = 0$). It is important to note that for pricing credit, it is not the asset drift itself, but rather the drift of the asset relative to the default boundary that is relevant. We assume that on average over time a firm issues more debt to maintain a steady level of leverage, or else pays dividends so that the debt has the same drift as the stock price. Given (2.25), to avoid arbitrage the same drift should be assigned to the asset value V, implying that the drift of the assets relative to the default barrier is indeed zero.

For given debt-per-share and estimation of recovery value, using (2.25) and (2.26), we obtain a closed form formula that involves only market observable parameters.[4]

Survival probability (Lardy, Finkelstein, Khuong-Huu and Yang (2000))

$$P(t) = \Phi\left(-\frac{A_t}{2} + \frac{\log(d)}{A_t}\right) - d \cdot \Phi\left(-\frac{A_t}{2} - \frac{\log(d)}{A_t}\right), \tag{2.28}$$

is expressed as a function of market observable parameters

$$d = \frac{S_0 + \bar{L}D}{\bar{L}D} e^{\lambda^2}, \tag{2.29}$$

$$A_t^2 = \left(\sigma_S^* \frac{S^*}{S^* + \bar{L}D}\right)^2 t + \lambda^2, \tag{2.30}$$

where

- S_0: initial stock price,
- S^*: reference stock price,
- σ_S^*: reference stock volatility,
- D: debt-per-share,
- $\bar{L}$: global debt recovery,
- λ: percentage standard deviation of the default barrier.

The debt-per-share D is based on financial data from consolidated statements. We first calculate all liabilities that participate in the financial leverage of the firm. These include the principal value of all financial debts, short-term and long-term borrowings and convertible bonds. Additionally, we include quasi-financial debts such as capital leases, under-funded pension liabilities or preferred shares. Non-financial liabilities such as accounts payable, deferred taxes and reserves are not included. Debt-per-share is then the ratio of the value of the liabilities to the equivalent number of shares. The equivalent number of shares includes the common shares outstanding, as well as any shares necessary to account for other classes of shares and other contributors to the firm's equity capital. In practice, the financial data used in the debt-per-share calculation

[4] Note that V_0 does not necessarily correspond to the real initial asset value, nor does σ necessarily correspond to the real asset volatility of the firm. Nevertheless, these simple expressions lead to an effective approximation for the distance to default in (2.24).

should be adjusted for recent events that are already priced in by the market. The details of the CreditGrades debt-per-share calculation are provided in Appendix B.

The mean ($\bar{L}$) and the percentage standard deviation (λ) of the global recovery L are estimated using the Portfolio Management Data and Standard & Poor's database (Hu and Lawrence (2000)). The database contains actual recovery data for approximately 300 non-financial U.S. firms that defaulted from 1987 to 1997. Defaulted instruments include bonds and bank loans. Based on the study of these historical data, $\bar{L}$ and λ are estimated to be 0.5 and 0.3, respectively. A lower λ is expected for the financial sector due to the sector specific government regulations.

2.3 Sensitivities to model parameters

2.3.1 Default barrier uncertainty

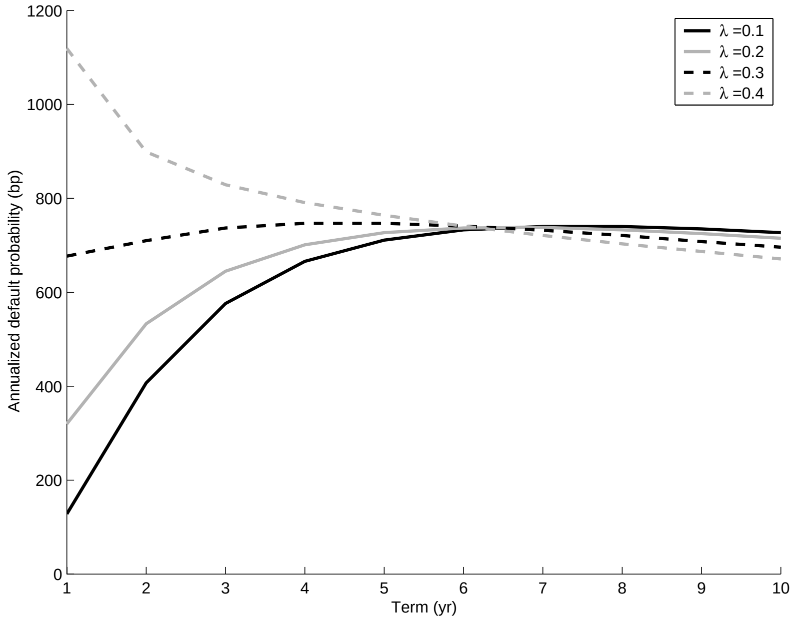
Examining (2.11), we see that longer term ($t > \lambda^2/\sigma^2$) survival probabilities are mostly determined by the distance to the default barrier η. For short-term maturities ($t < \lambda^2/\sigma^2$) the main driver of default probability is the uncertainty, represented by λ, of the actual level of the barrier. For all but the most leveraged cases, higher values of λ correspond to higher probabilities of short-term default. For higher quality firms, because the chance of moving to the default barrier in the short term by pure diffusion is very small, the short-term default probabilities are almost entirely driven by λ.

Define $p(t) = -\log(P(t))/t$ to be the annualized probability of default. The shape of the term structure of default probabilities is determined by the initial distance to the barrier (d), the volatility (σ) and the standard deviation of the barrier (λ), with the short end of the term structure primarily driven by λ and d. A wide variety of term structures can be achieved through different combinations of the parameters. Figure 2.2 illustrates the impact of λ on the annualized probability of default $p(t)$. As expected, higher uncertainty of debt recovery leads to higher short-term probability of default. Figure 2.2 also shows that the term structures move from upward sloping to inverted as the uncertainty of debt recovery increases.

2.3.2 Equity price and volatility

The CreditGrades model allows us to characterize sensitivities of a credit derivative instrument to changes in market prices analogously to equity derivative instruments. Given a CDS spread, the implied asset (or equity) volatility is the value of σ (or σ_S) that, when plugged into (2.28) and (2.15), recovers the actual CDS spread. The implied volatility can be used similarly to an implied volatility in the equity derivatives market: to communicate a pricing level, to calculate sensitivities or to evaluate the relative pricing of similar derivatives.

Figure 2.2: Impact of λ on Default Probability ($V_0/(\bar{L}D) = 2$, $\sigma = 0.25$)



Sensitivities to the equity price and volatility can be characterized by the CreditGrades delta, gamma and vega. As an illustration, consider a USD 1,000 notional 5-year credit default swap on a firm with a debt-per-share of USD 30 and asset specific recovery of 30 percent. The CreditGrades delta represents the sensitivity of the market value of a CDS to changes in the firm's equity price, and can be interpreted as the number of shares of the equity required to offset small price moves.[5] Figure 2.3 shows the relationship between delta and the equity spot price. The number of shares needed to hedge the CDS increases as spot price drops.

The CreditGrades gamma represents the change in delta with respect to spot price moves. Figure 2.4 shows that CDS gamma increases as a stock price decreases. Further, higher levels of asset volatility dampen the value of gamma for default swaps on distressed credits.

Finally, the CreditGrades asset vega represents the dollar change in the CDS value per 1 percent move in asset volatility. The asset vega can be converted to an equivalent equity vega using (2.26). The relationship between vega and spot price is shown in Figure 2.5. In particular, for default swaps on a credit with high stock price, vega increases with the asset volatility, indicating the rise in downside risk with increasing uncertainty. Not surprisingly, all of the sensitivities shown in Figures 2.3 to 2.5 are quite similar to those of an equity put option.

[5] In all cases here, we consider the sensitivities of the contingent leg of the CDS only, since this is where most of the sensitivity to equity lies. In other words, we assume in all cases that there is no CDS premium being paid.

Figure 2.3: Delta versus Spot Price for a 5-year CDS with $\bar{L} = 0.5$, $\lambda = 0.3$, $D = 30$, and $R = 0.3$

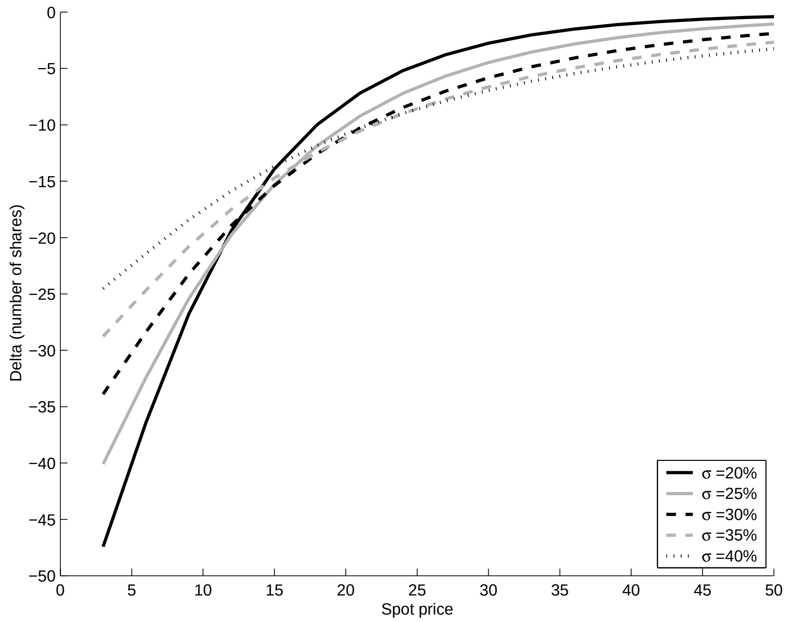


Figure 2.4: Gamma versus Spot Price for a 5-year CDS with $\bar{L} = 0.5$, $\lambda = 0.3$, $D = 30$, and $R = 0.3$

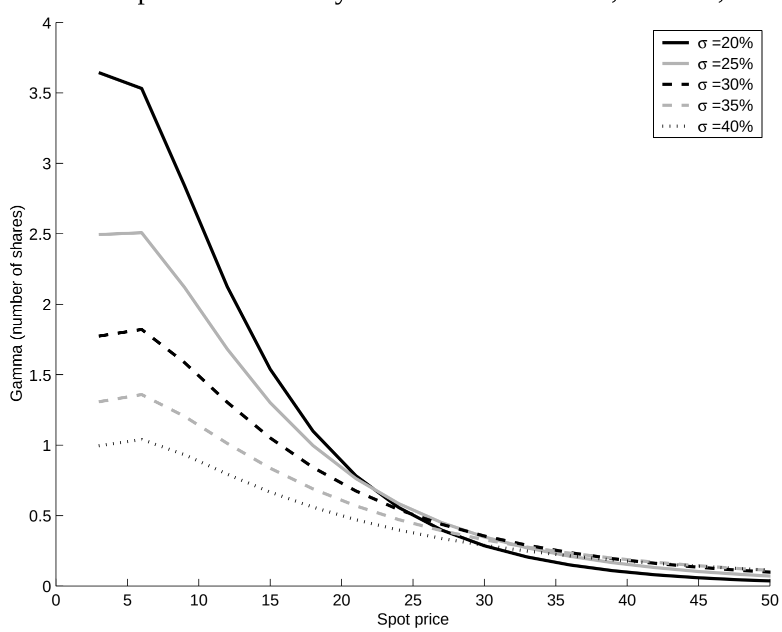
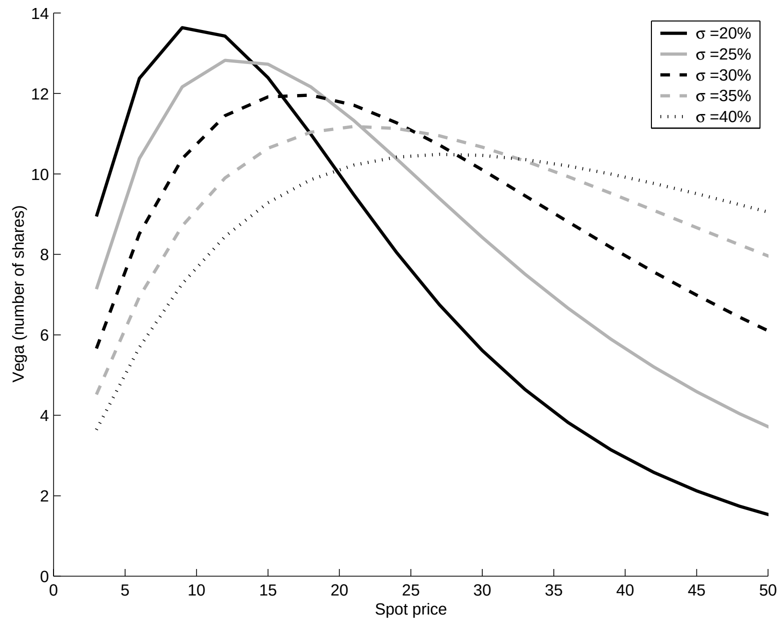


Figure 2.5: Vega versus Spot Price for a 5-year CDS with $\bar{L} = 0.5$, $\lambda = 0.3$, $D = 30$, and $R = 0.3$



Chapter 3

Empirical Results

In this chapter, we address model implementation and testing. Our first step is to choose an estimator for asset volatility. In order that the CreditGrades™ model be applicable for a wide variety of firms, we restrict our search to estimators that rely only on equity price history and choose an estimator that best matches actual Credit Default Swap (CDS) spreads. With the estimator chosen, we investigate the performance of the CreditGrades model as a tool for ranking credits and for tracking moves in market CDS spreads. Finally, we examine a number of test cases.

The data used in this chapter includes mid-market quotes on 5-year CDS spreads from the period May 2000 through August 2001. The quotes were taken for 122 U.S. firms, representing a wide range of credit quality and industrial sectors (see Table 3.1). Bond-specific recovery rates for each spread were taken from the JPMorgan database. In total, 6,194 quotes are used in the analysis. Historical stock prices and closing 5-year LIBOR rates are from DataMetrics and debt-per-share is calculated as detailed in Appendix B. Throughout the chapter, we assume that the standard deviation λ of the default barrier is 0.30 and that the global recovery rate $\bar{L}$ is 0.50.

3.1 Asset volatility estimator

In this section, we examine estimators for the implied asset volatility parameter σ. Recall that the CreditGrades implied asset volatility is the level of σ for which the CreditGrades model recovers the market CDS price. Thus, our task is to identify an appropriate estimator $\hat{\sigma}$ for the implied asset volatility.

We begin with the implied asset volatility for each quoted CDS spread. Historically, the implied asset volatility has been quite stable. Figure 3.1 plots for each firm the individual observations of the implied asset volatility versus the historical average implied volatility for that firm. The stability of the implied asset volatility is evident in the plot, with only four firms[1] showing variations of more than 10 percent.

Table 3.1: Description of Data

S&P credit rating		#firms	Sector break-down		#firms
AAA	-	1	Basic materials	-	12
AA	-	9	Communications	-	19
A	-	38	Consumer, cyclical	-	24
BBB	-	53	Consumer, non-cycl.	-	22
BB	-	17	Energy	-	12
B	-	3	Financial	-	1
CCC	-	1	Industrial	-	19
			Technology	-	7
			Utilities	-	6

Next, given historical equity prices, several equity volatility estimates $\sigma_S^{(t)}$ are calculated for each observed CDS quote, where t is the observation date. We calculate $\sigma_S^{(t)}$ based on window sizes of 252, 500, 750, 1000 and 1250 days of daily equity returns. In addition, we examine the exponentially weighted moving average (EWMA) volatility estimate with a decay factor of 0.94, a typical estimate for short term risk management uses. The estimate of asset volatility at time t is then calculated by applying the gearing ratio to $\sigma_S^{(t)}$:

$$\hat{\sigma}^{(t)} = \sigma_S^{(t)} \cdot \frac{S_t}{S_t + \bar{L}D} \tag{3.1}$$

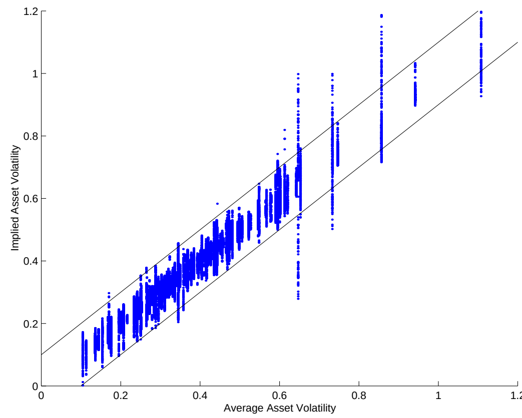
where S_t is the current stock price, D is the debt-per-share and $\bar{L}$ is the average global recovery rate.

Figure 3.2 shows the results for the various estimators. In the scatterplots, each point represents a single CDS quote. For each quote, we plot the estimated asset volatility versus the CreditGrades implied asset volatility. We observe that volatility estimates based solely on recent data (especially the EWMA estimator) are inappropriate. As we consider longer historical periods, the volatility estimators improve, with estimates based on 750 or 1000 days of observations being optimal. These results are sensible since our goal is to estimate a long-dated volatility.

We present the same scatterplot for the 1000-day estimator in Figure 3.3, with the individual points identified by credit rating. The volatility estimator appears strong for the entire spectrum of credit quality. However, there is a slight tendency for the model to underestimate volatility for the best quality firms, the best performance is achieved for the speculative grade firms. Both of these observations are expected: the relationship between equity and credit is strongest for the high yield firms, while for the best quality names there is a significant skew in volatility toward the deep out-of-the-money cases that a default event would represent.

[1] Excite@home, Exodus Communications, Nextlink Communications and Williams Communications.

Figure 3.1: Implied Asset Volatility versus Average Implied Asset Volatility. All firms, all dates.



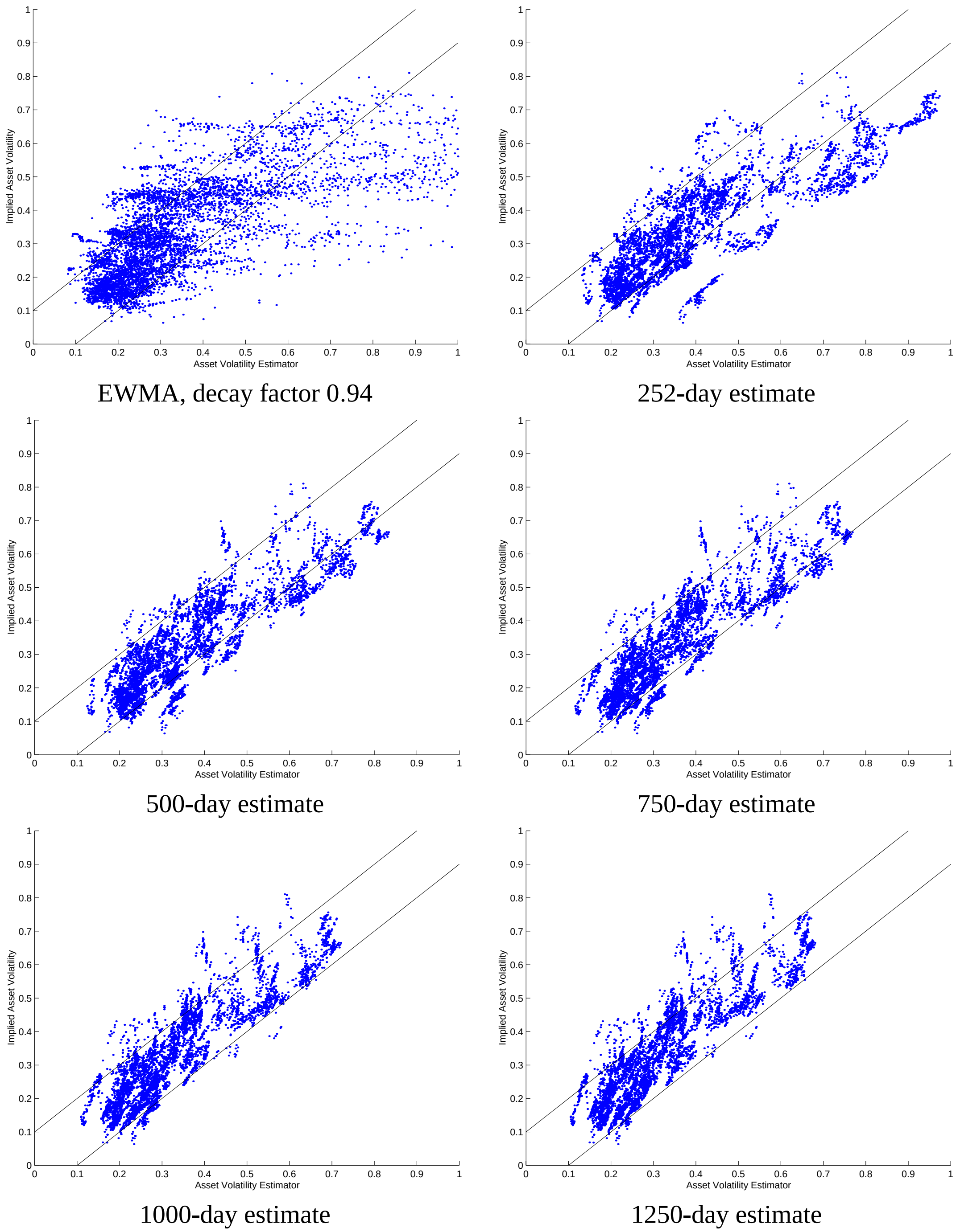
Overall, the 1000-day window appears to work best, with 88 percent of the estimates falling within 10 percent of the true value. Table 3.2 lists the firms that had 20 or more data points where the volatility estimation error was greater than 10 percent. In total, this represents 503 data points. Of these names, four firms had errors greater than 15 percent, representing 148 data points.

Next, we separate the data into two time periods. Figure 3.4 plots the implied asset volatility against the 1000-day volatility estimator, separating quotes from before and after January 1, 2001. From this figure, it appears that the quality of the historical estimator is invariant to time.

Finally, we examine the 1000-day estimator on four different days, including one (November 15, 2001) that does not fall in our sample data period. The implied and estimated asset volatility for each firm are shown in Figure 3.5. The results again show that a simple 1000-day estimate is a good approximation to the true volatility.

In summary, our research shows that a simple measure of volatility based on the last 1000 days of equity returns produces good estimates of the volatility implied by the 5-year CDS quotes. The choice of a long-dated estimator is not surprising since we have shown that the true asset volatility for a firm is quite stable through time. The 1000-day historical volatility estimator is robust across a broad range of industrial sectors and credit ratings.

Figure 3.2: Implied Asset Volatility versus Historical Volatility Estimators



EWMA, decay factor 0.94

252-day estimate

500-day estimate

750-day estimate

1000-day estimate

1250-day estimate

Figure 3.3: Historical versus Implied Volatility by Credit Quality

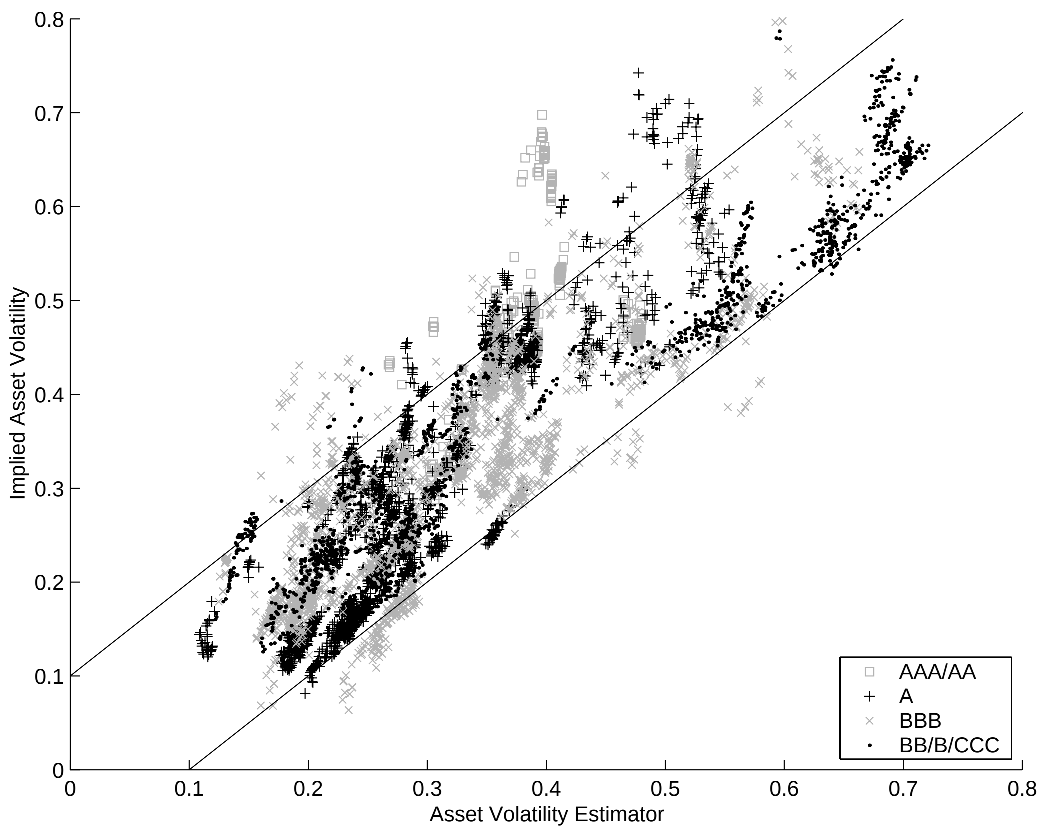
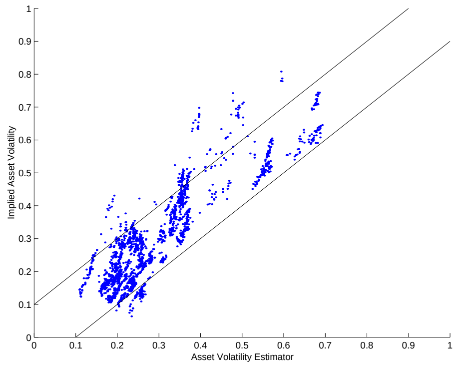


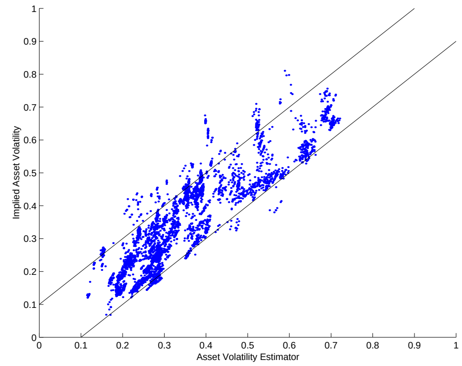
Table 3.2: Outliers for 1000-day Estimator (†Indicates firms with errors over 15 percent.)

AMR Corp.	J.C. Penney Co.†
Boston Scientific Corp.	Eli Lilly & Co.
CMS Energy Corp.	Schering Plough Corp.†
Coca-Cola Enterprises	Viacom
Corning Inc.†	Wal-Mart Stores
Exxon Capital Corp.†	

Figure 3.4: Scatterplot of Implied Asset Volatility versus 1000-day Historical Estimator

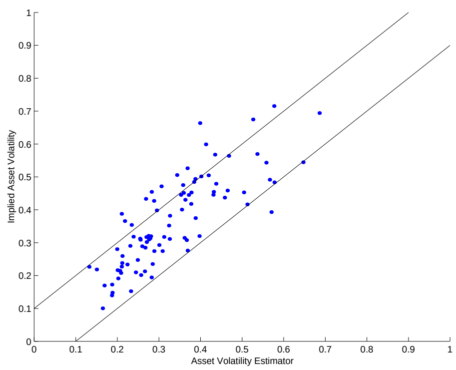


Before January 1, 2001

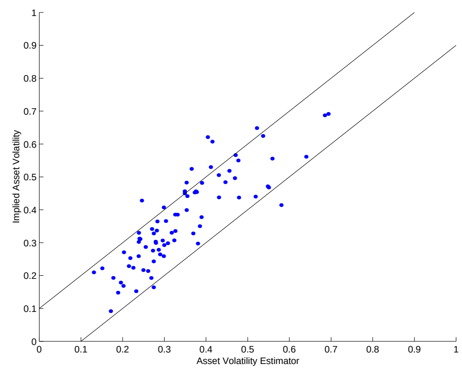


After January 1, 2001

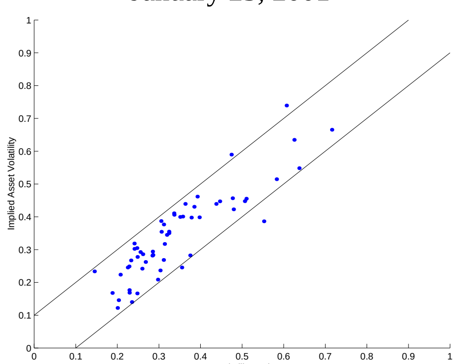
Figure 3.5: Scatterplot of Implied Asset Volatility versus 1000-day Historical Estimator



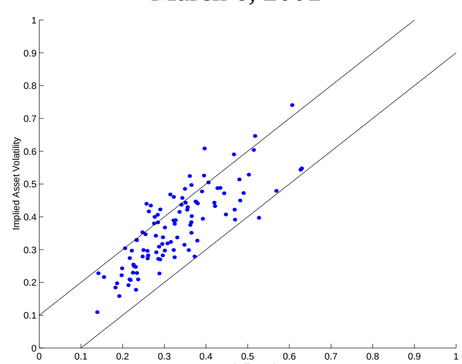
January 25, 2001



March 6, 2001



August 10, 2001



November 15, 2001

3.2 Relative credit risk assessment

To this point, our emphasis has been on the accuracy of the CreditGrades model as an indicator of actual CDS spread levels. Another important application of the CreditGrades model is as a means to rank credits on a relative basis. While agency credit ratings may be used for the same purpose, the incorporation of observable parameters and market data is attractive. In this section, we examine the performance of the CreditGrades model in ranking credits and compare this performance to other models.

We will examine data for 107 firms on two dates: January 30, 2001 and November 15, 2001. On each date, we will compare the 5-year default probability derived from four sources:

1. The CreditGrades model with 1000-day historical volatility estimator (referred to simply as CreditGrades)
2. The CreditGrades model with equity volatility set to the at-the-money equity volatility for the longest tenor available in the DataMetrics™ database (referred to as Implied Volatility)
3. The Moody's RiskCalc™ model
4. The risk-neutral default probabilities implied by actual CDS spreads

The market ranking of the credits implied by CDS spreads will be considered as the "correct" ranking; the other models will be assessed on their ability to rank credits similarly to the market.

3.2.1 Cumulative Accuracy Profiles

One method to examine a model's ability to accurately rank credit risks is through a Cumulative Accuracy Profile (CAP). In default studies, it is typical to create CAP's by plotting the proportion of defaulting firms that are among the riskiest x firms according to the model across all values of x. Since we are comparing to market spreads rather than to actual defaults, the CAP's here are slightly different.

In our CAP plots, the firms are first ranked based on the market-implied cumulative default probabilities. The same firms are then ranked based on one of the other model's probabilities of default. Let N be the total number of firms in the sample. A point on the CAP is constructed as follows:

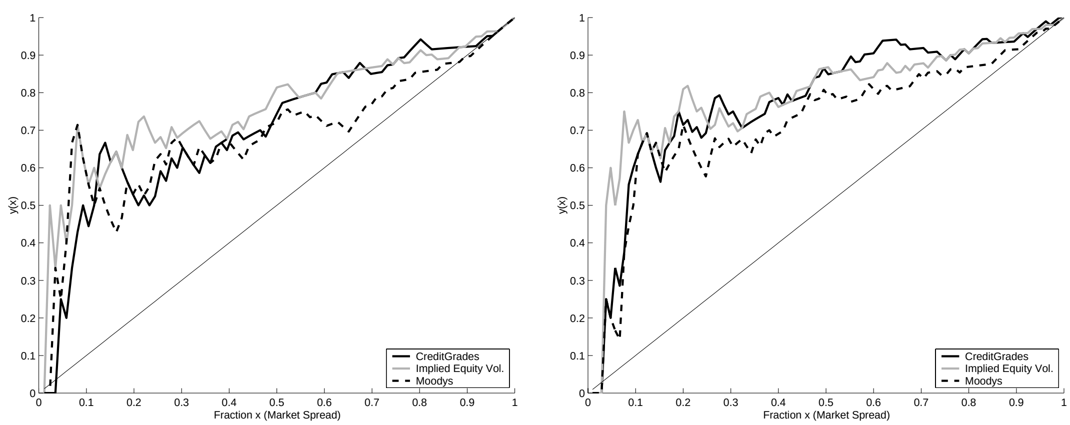
1. First, select a value x between zero and one and identify the riskiest $x \cdot N$ firms according to the market-based default probabilities.
2. Next, compute the proportion $y(x)$ of the $x \cdot N$ riskiest firms according to the model to be examined that are also among the market's $x \cdot N$ riskiest firms.

For example, say $N = 100$ and $x = 10\%$. We then count how many of the 10 riskiest firms according to the model are among the 10 riskiest firms according to the market. If this number is eight, then the y value corresponding to $x = 10\%$ is $y(x) = 80\%$.

If the model and market rank the firms identically, then $y(x)$ will be equal to 100% for every value of x. On the other hand, if the model produces no more than a random shuffling of the firms, then we expect $y(x) = x$ for every value of x.

The CAP plots are presented in Figure 3.6. The results show that the CreditGrades ranking has significant information on how the market actually ranks credit risk. Across the three models, it appears that the implied volatility model performs slightly better in ranking the worst credits (values of x less than 0.1) while the historical volatility model has a comparative advantage in ranking among the better credits (values of x greater than 0.5). As we will see in Chapter 4, when the stock moves suddenly, it is often useful to examine the implied volatility to take into consideration the full impact of the move.

Figure 3.6: Cumulative Accuracy Profiles



January 30, 2001 November 15, 2001

3.2.2 Mismatch distribution

Next, we compare the market and the respective models by their assignment of the firms into risk buckets. In particular, we label the firms with the decile into which they fall under each model. Thus, the riskiest 10 percent of the firms according to the market receive a risk score of 10, the next riskiest 10 percent a risk score of 9, and so on. We assign risk scores for the models similarly. We then examine the differences between the market and model risk scores.

We present histograms of the risk score differences in Figure 3.7. We see that for each of the methodologies, most of the firms have a difference between -1 and +1, though there are some large discrepancies. We list in Tables 3.4 and 3.3 the firms for which the model risk scores differ from the market by more than 3. All three models appear to perform better on the November data, with the CreditGrades and implied volatility models performing slightly better than Moody's RiskCalc. Further, the most significant outliers tend to be cases where the models impute a high default probability to a firm that is ranked much better by the market.

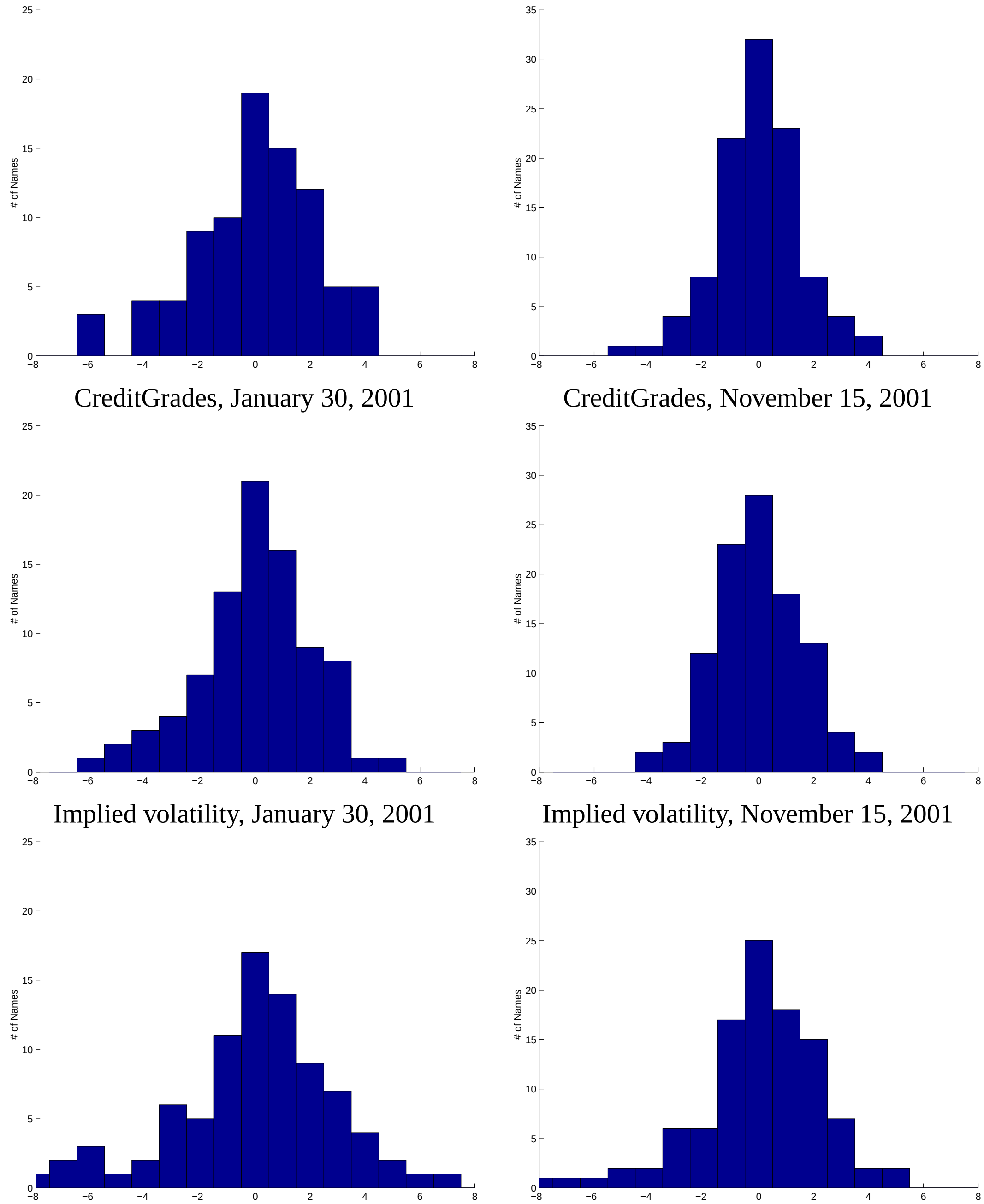
Table 3.3: Risk Score Error — November 15, 2001

Model	Firm	Market Risk Score	Model Risk Score	Error
CreditGrades	Coca-Cola Enterprises	2	7	-5
CreditGrades	Deere & Co.	4	8	-4
CreditGrades	Eastman Kodak Co.	9	5	4
CreditGrades	Tricon Global Rest.	8	4	4
Implied volatility	Albertsons Inc.	5	1	4
Implied volatility	Coca-Cola Enterprises	2	6	-4
Implied volatility	Public Service Entrp.	1	5	-4
Implied volatility	Tricon Global Rest.	8	4	4
Moody's RiskCalc	Avon Products	1	8	-7
Moody's RiskCalc	Black & Decker	4	9	-5
Moody's RiskCalc	Ball Corp.	8	4	4
Moody's RiskCalc	Coca-Cola Enterprises	2	8	-6
Moody's RiskCalc	Clear Channel Comm.	8	3	5
Moody's RiskCalc	Cendant Corp.	10	6	4
Moody's RiskCalc	Deere & Co.	4	8	-4
Moody's RiskCalc	Gillette Co.	1	6	-5
Moody's RiskCalc	Mattel Inc.	7	2	5
Moody's RiskCalc	Public Service Entrp.	1	10	-9
Moody's RiskCalc	Rohm & Haas Co.	3	7	-4

Table 3.4: Risk Score Error — January 30, 2001

Model	Firm	Market Risk Score	Model Risk Score	Error
CreditGrades	Anadarko Petrol. Corp.	3	7	-4
CreditGrades	Bausch & Lomb Inc.	10	6	4
CreditGrades	Coca-Cola Enterp.	1	7	-6
CreditGrades	Corning Inc.	9	5	4
CreditGrades	CVS Corp.	5	1	4
CreditGrades	Deere & Co.	3	9	-6
CreditGrades	General Motors Corp.	5	9	-4
CreditGrades	Kinder Morgan Inc.	4	8	-4
CreditGrades	Lockheed Martin Corp.	4	8	-4
CreditGrades	Sprint Corp.	9	5	4
CreditGrades	VF Corp.	7	3	4
CreditGrades	Weatherford Intl. Inc.	4	10	-6
Implied volatility	Anadarko Petrol. Corp.	3	8	-5
Implied volatility	Archer Daniels Midland	1	5	-4
Implied volatility	Coca-Cola Enterp.	1	7	-6
Implied volatility	Conagra Inc.	6	2	4
Implied volatility	Deere & Co.	3	7	-4
Implied volatility	Hewlett Packard	2	6	-4
Implied volatility	Mattel Inc.	10	5	5
Implied volatility	Phelps Dodge Corp.	3	8	-5
Moody’s RiskCalc	Avon Products	4	10	-6
Moody’s RiskCalc	Boston Scientific Corp.	9	5	4
Moody’s RiskCalc	Clear Channel Commun.	8	3	5
Moody’s RiskCalc	Clorox Co.	1	8	-7
Moody’s RiskCalc	Coca-Cola Enterp.	1	9	-8
Moody’s RiskCalc	ConEd. Co. of NY	6	1	5
Moody’s RiskCalc	Cox Communications	8	4	4
Moody’s RiskCalc	Deere & Co.	3	9	-6
Moody’s RiskCalc	Diamond Offshore Drilling Co.	5	1	4
Moody’s RiskCalc	Dow Chemical Co.	3	8	-5
Moody’s RiskCalc	E.I. Du Pont De Nemours & Co.	1	8	-7
Moody’s RiskCalc	General Motors Corp.	5	9	-4
Moody’s RiskCalc	Mattel Inc.	10	3	7
Moody’s RiskCalc	Procter & Gamble Co.	1	7	-6
Moody’s RiskCalc	Rohm & Haas Co.	4	8	-4
Moody’s RiskCalc	VF Corp.	7	1	6
Moody’s RiskCalc	Weyerhaeuser Co.	7	3	4

Figure 3.7: Mismatch Distribution of Models versus Market. 107 Firms.



CreditGrades, January 30, 2001

CreditGrades, November 15, 2001

Implied volatility, January 30, 2001

Implied volatility, November 15, 2001

Moody's RiskCalc, January 30, 2001

Moody's RiskCalc, November 15, 2001

3.2.3 Measures of association

Finally, we consider more rigorous measures of the association between the market- and model-based default probabilities. One standard measure is a simple correlation statistic. However, correlation can be overly influenced by outlier data. Further, the default probabilities in our sample range over several orders of magnitude. The correlation statistic is then likely to be dominated by the firms with the highest default probabilities, and high credit quality firms given little weight.

To address these concerns, we also examine a rank correlation statistic, Kendall's tau. Kendall's tau for two random variables X and Y is defined as

$$\tau = \mathbf{P}\{(X_2 - X_1)(Y_2 - Y_1) \geq 0\} - \mathbf{P}\{(X_2 - X_1)(Y_2 - Y_1) < 0\}, \qquad (3.2)$$

where (X_1, Y_1) and (X_2, Y_2) represent two independent realizations from the joint distribution of X and Y. In our context, an interpretation of the first term in (3.2) is the probability that for a randomly chosen pair of firms, both the market and model will rank the firms in the same order. The second term represents the probability that the market and model disagree on the ranking.

For a sample of bivariate observations, (X_i, Y_i), for $i = 1, ..., k$, a non-parametric estimate for Kendall's tau is

$$\hat{\tau} = \frac{2}{k(k-1)} \sum_{i<j} \text{sign}[(X_i - X_j)(Y_i - Y_j)]. \qquad (3.3)$$

Two series that are identical will have a Kendall's tau of 1 while a Kendall's tau of 0 will indicate no association. Note that Kendall's tau is insensitive to large discrepancies in the data and to a large range of scales.

We present the measures of association for the three models with the market in Table 3.5. Again, we see that all of the models perform better on the November sample, and that the two CreditGrades implementations track the market better than the Moody's RiskCalc model. This last point is to be expected, since Moody's RiskCalc is trained to actual default data and outputs an objective default probability while the CreditGrades models represent a market-based probability.

In all, the results are quite encouraging. For the November data, the correlation between the market and CreditGrades default probabilities is over 80 percent. This figure drops for the January data, but is still over 60 percent. Examining the more robust measures of rank correlation, we see closer results across the two samples, with the probability of a correct ordering of a random pair of firms ranging from 75 percent to 85 percent for both CreditGrades implementations.

Table 3.5: Comparison of Market versus Model Ranking — Kendall's Tau and Correlation

Volatility Estimator	Kendall's Tau	Prob. of correct ranking	Correlation
January 30, 2001			
CreditGrades	0.522	0.761	0.602
Implied volatility	0.567	0.784	0.748
Moody's RiskCalc	0.358	0.679	0.576
November 15, 2001			
CreditGrades	0.667	0.834	0.816
Implied volatility	0.653	0.827	0.846
Moody's RiskCalc	0.502	0.751	0.629

Chapter 4

Applications and Case Studies

When applying any quantitative model, it is crucial to be aware of the model's inputs and assumptions. In this chapter, we will examine a number of applications of the CreditGrades™ model and in so doing pay particular attention to the CreditGrades inputs.

CreditGrades can be used by a wide range of credit market practitioners:

- Bank loan book — Bank loan officers and loan portfolio managers can use CreditGrades to monitor and quantify whether credit risk is rising in their portfolios. Equity market-related information may lead a bank to increase its pricing on new loans, or to decide to sell or hedge a loan sooner than it might have otherwise done.
- Risk management — Risk managers can use CreditGrades as an alternative source of information from the equity markets about potential credit risk in a credit portfolio.
- Corporate bond investors — Since CreditGrades quantifies the implied credit risk of a bond based on equity market parameters investors can potentially spot emerging credit problems and identify either technical or fundamental factors that may be driving bond spreads. For trading-oriented investors, this may give rise to interesting relative value and cross market trading opportunities.

As we have described previously, using the structural model framework, CreditGrades derives the level of credit risk associated with a given firm based on the firm's capital structure, stock price and volatility. The model expresses credit risk in terms of a default probability and a CreditGrades score, which corresponds to a credit spread to LIBOR or a default swap premium.

As we have demonstrated, there is a strong relationship between CreditGrades scores and market credit spreads over time and across industry sectors. But we stress that CreditGrades is not a credit pricing model per se. Indeed, cases where the CreditGrades score and market credit spreads are within 10 to 20 percent of each other represent a relatively strong relationship between the equity and credit market views of a firm.

The variation between CreditGrades and the market is unavoidable. It is important to recognize what factors are included and not included in the model:

- CreditGrades does not include any inputs that reflect bond market supply and demand technicals, liquidity or new issue activity, all of which are likely to be largely independent of equity prices.

- Applied to the default swap market, CreditGrades does not include inputs to capture technical factors that affect the basis between the cash and default swap sectors. Such factors include demand related to a surge in lending or syndicated loan market activity, heavy supply of protection due to portfolio swap activity or bonds trading at a significant premium or discount to par. CreditGrades also cannot capture the significant positive basis often seen when the reference obligation is a convertible bond.

- The long-term historical volatility estimator used in CreditGrades is robust in reasonably stable periods. However, when a firm's stock or credit moves suddenly, the historical volatility can lag true market levels. In these cases, it is constructive to examine implied volatility levels.

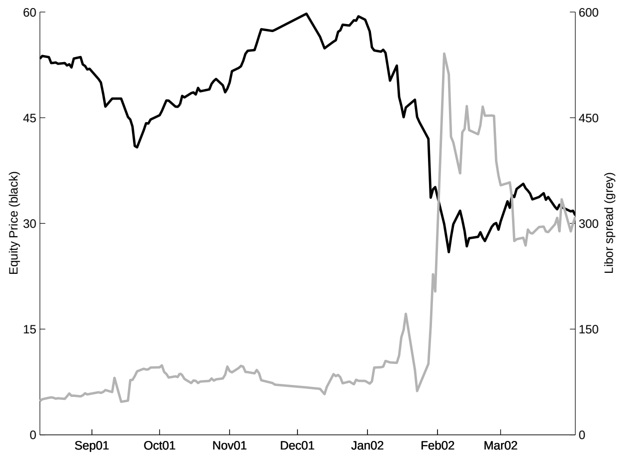
- The debt-per-share data used in CreditGrades is obtained from Bloomberg based on standard quarterly or annual filings. If a firm changes its capital structure between reporting period, or if it has a significant finance subsidiary included in its consolidated statements, the CreditGrades score will be affected.[1] Users must be prepared to review the standard inputs into CreditGrades to make sure they are up to date and representative of the firm and the markets.

Ultimately, the true value in an automated model such as CreditGrades goes beyond its ability to predict absolute credit spreads, and lies in its ability to identify firms where the credit and equity markets may be developing divergent views of the credit risk outlook. Such situations may indicate emerging risks or opportunities, or else a need to adjust or update one of the inputs.

In the sections below, we provide examples of how CreditGrades can be used to analyze potential transactional opportunities or to monitor changes in the credit riskiness of particular firms.

[1]CreditGrades has been calibrated for use with industrial firms, broadly defined to include standard sectors such as basic industry, automotive, retail, technology, consumer and telecommunications. The universe does not include bank and finance firms. These entities are very highly leveraged, which results in very high CreditGrades scores. Actual spreads for these sectors are much tighter for several reasons. Banks benefit from government oversight and policies that provide for conservatorship as an alternative to bankruptcy. Both banks and finance firms also have large quantities of secured liabilities such as repurchase agreements, so their effective leverage ratio is lower than implied by standard debt-per-share calculations.

Figure 4.1: Tyco International



4.1 Tyco International

From the mid-1990s through much of 2001, Tyco International (TYC) was a market darling. This changed abruptly following Enron's demise, when investors and the SEC turned their focus to TYC's accounting practices. As shown in Figure 4.1, between December 2001 and February 2002, TYC stock fell from near USD 60 to about USD 30, and its credit spread[2] widened from 110bp to over 500bp. TYC default swaps traded 50bp to 200bp wider than bonds in technical trading.

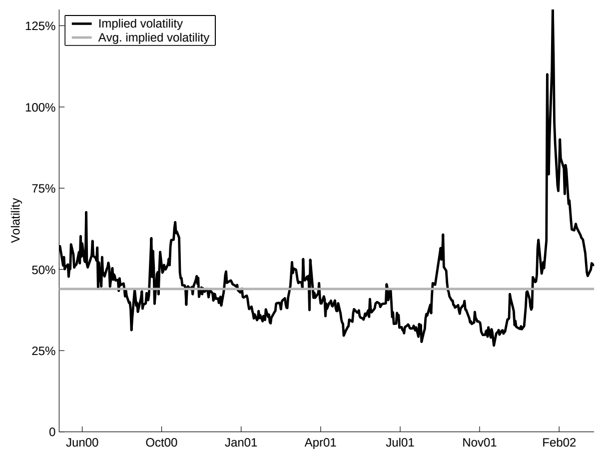
Prior to Enron's demise, when TYC stock was trading near USD 55, the TYC CreditGrades score was about 100bp, roughly in line with the LIBOR spread of bonds. As TYC's stock price fell to USD 30 in early 2002, the CreditGrades score rose to about 230 bp while the bond spread widened considerably more, to over 500bp.

The key explanatory factor for the relatively low CreditGrades score was the volatility input. When TYC ran into problems, implied equity volatility shot up from about 43 percent to the 70–80 percent range. However, based on the CreditGrades standard 1000-day historical volatility estimate, the CreditGrades asset and equity volatility inputs remained steady near 34 percent and 49 percent, respectively. Adjusting the CreditGrades asset volatility input upward to 60 percent (corresponding to an equity volatility of about 75 percent), we see a CreditGrades score of 580bp, roughly in line with credit spreads at the height of TYC's troubles.

It is apparent from this analysis that TYC bonds and protection were trading in line with equity volatility. A review of TYC's equity volatility over time (see Figure 4.2) shows that it has been variable, but also has

[2] The TYC credit spread is measured versus LIBOR for the 5.8 percent 2006 bond.

Figure 4.2: Tyco International volatility



tended to revert to its long-run mean. The question in February was whether TYC would survive. Investors who believed that TYC would ultimately prevail could have used the insights from CreditGrades to justify buying bonds or selling protection, thereby capitalizing as implied equity volatility receded to more normal levels. As the figures illustrate, credit spreads did tighten to about 310bp as equity volatility fell to near 50 percent. At the same time, the CreditGrades score fell to 275bp, well within the acceptable error range for the model. The CreditGrades model indicates that for TYC spreads to tighten to pre-crisis levels, either the equity price must rise to about USD 55 or equity volatility will have to fall to about 38 percent.

CreditGrades allows an investor to analyze and quantify these kinds of relationships among credit spreads, stock prices and equity volatility, especially when a company's credit spreads and equity prices are trading roughly in line with each other.

4.2 WorldCom

In September 2001, WorldCom (WCOM) equity was trading in the USD 13–14 range and CDS protection mid-market levels were 165bp to 175bp. The CreditGrades model score was 225bp to 250bp. This could have indicated that either the equity had become a bit cheap relative to the credit market, or that the credit market was lagging the equity market. In either case, the CreditGrades score was well within what we would consider to be an acceptable error range for the model.

At the time, there was growing interest in "hidden" leverage or off-balance sheet debt. Even though Enron's woes were only beginning and few if anyone outside the firm had any insight into the full extent of its off-

Table 4.1: Hidden Leverage Ratio Analysis of Telecommunications Firms

	Debt-to-Market Capitalization Ratio		
	Balance Sheet	Comprehensive	Multiple
WorldCom	46%	90%	1.9
Sprint	17%	33%	1.9
Citizen's Communications	70%	96%	1.4
Qwest	43%	50%	1.2
AT&T	40%	64%	1.6
Corning	24%	30%	1.3

balance sheet financing, there were growing concerns about hidden debt or leverage at telecommunications and technology companies that had borrowed heavily during the 1997–1999 bubble.

We analyzed recent quarterly filings for a number of telecommunications and technology firms. In each case, we examined the ratio of debt-to-market capitalization using only balance sheet debt (the Balance Sheet ratio), and then including disclosed off-balance sheet obligations as well (the Comprehensive ratio). The results are presented in Table 4.1. For WCOM, the Comprehensive ratio was nearly double the Balance Sheet ratio, a much higher difference than that for the other firms. This was largely due to off-balance sheet operating leases, a legacy of WCOM's growth-by-acquisition strategy. Sprint's Comprehensive ratio is also a high multiple of its Balance Sheet ratio, but its overall level of debt is relatively low.

When the WCOM debt-per-share input is increased to reflect off-balance sheet debt, the CreditGrades score rises roughly linearly. The question is whether the credit markets would become concerned about WCOM's true leverage ratios and adjust credit spreads upward. Whether this situation presented a relative value trading opportunity is a judgment call. Some investors and analysts might conclude that information about the operating leases is already in the market (since it is disclosed in the financial statements); others might conclude that the information was partially in the market, implying a smaller effective differential between balance sheet and comprehensive leverage.

Assuming one believed that the September 2001 price levels revealed a relative value opportunity, and that credit spreads would widen and/or stock prices would fall, there were several ways to express this view:

1. Buy protection outright — the cost of carry is relatively high.
2. Short WCOM stock outright — depends on the availability of WCOM stock lenders.
3. Buy WCOM puts — liquidity in OTC puts is limited, cost may be relatively high.
4. Buy protection, buy equity or buy calls.

5. Sell protection, sell equity short or buy puts.

We will consider alternative 4 - buy protection and buy equity. Since we expect credit spreads to widen sharply, the hedge ratio in this trade will be heavily skewed to protection. The equity position is a hedge against a scenario where the equity markets and credit markets perform much better than expected, perhaps due to a merger announcement or to a sudden change in the outlook for the economy and telecommunications companies.

The CreditGrades model gives us an initial indication of an appropriate hedge ratio to use in our position. Assuming a base case with a stock price of USD 13, an asset volatility of 40 percent and a debt-per-share of USD 9, we have a CreditGrades score of 249bp. Moving the stock price to USD 14 while holding the other parameters constant, we obtain a CreditGrades score of 224bp, or a tightening of 25bp. The spread duration of a 5-year CDS is approximately four years, meaning that the present value sensitivity of the CDS to a one dollar move in the stock price is about 100bp. The change in stock price from USD 13 to USD 14 is 7.7 percent, suggesting a hedge ratio of 7.7:1. Of course, there are basis risks between the actual CDS spread and the CreditGrades spread as well as non-linearity in the relationship between the equity price and CreditGrades spread. In addition, an investor may want to overhedge if they are generally bearish on WCOM and underhedge otherwise. In our example, we consider hedge ratios of 10:1 and 5:1.

We simulate the performance of the trade based on the two hedge ratios (see Table 4.2). The last two columns present the NPV of the 10:1 and 5:1 hedged position. As of the end of March the NPV for the 10:1 strategy was USD 794,813, based on a CDS notional amount of USD 10 million and a USD 1 million cash outlay for the equity position. This translates to a 79 percent return on investment. If the equity position had been financed, the return on investment would have been even higher. The 5:1 hedge ratio strategy performed well

Table 4.2: Simulated Trading Strategy. On 15-Sep-01, purchase CDS protection on USD 10 million and purchase equity hedge.

	Protection (bp)	Stock	Cum. CDS Fee	CDS NPV	Equity NPV 10:1	Equity NPV 5:1	Position P&L 10:1	Position P&L 5:1
15-Sep-02	175	13.00	–	–	1,000,000	2,000,000	–	–
30-Sep-01	162	15.04	-7,292	-55,098	1,156,923	2,313,846	94,533	251,456
31-Oct-01	207	13.45	-21,875	133,264	1,034,615	2,069,231	146,004	180,620
30-Nov-01	167	14.54	-36,458	-33,840	1,118,462	2,236,923	48,163	166,625
31-Dec-01	170	14.08	-51,042	-21,125	1,083,077	2,166,154	10,910	-93,987
31-Jan-02	300	10.05	-65,625	502,182	773,077	1,546,154	209,634	-17,289
28-Feb-02	350	7.52	-80,208	689,738	578,462	1,156,923	187,991	-233,547
31-Mar-02	550	6.74	-94,792	1,371,143	518,462	1,036,923	794,813	313,274

through November 2001, but the subsequent fall in the stock price more than offset the gains from wider protection levels.

As of April 2002, WCOM equity is trading near USD 6 per share, implied equity volatility is above 90 percent and 5-year CDS protection is quoted near 850bp. The WCOM CreditGrades score is about 550bp. However, this is based on the historical equity volatility of only 45 percent. If we adjust the equity volatility closer to the implied level of 90 percent, the CreditGrades score widens to 1200bp. From here, will the actual volatility revert to the mean, and lead to sharply tighter spreads? Or is the equity market signaling further deterioration in WCOM credit spreads? CreditGrades can provide analytic insights into these issues and quantify an analyst's expectations, but effectively applying these results still requires human insight and knowledge of the broader credit and market trends.

4.3 Enron

In the remaining sections, we examine the use of the CreditGrades score as a monitoring device. For four firms, we track the benchmark CreditGrades 5-year default probability (DP) along with the default probability implied by the firm's Standard & Poor's credit rating. The Standard & Poor's default probabilities are determined by the long-run historical default rate for each rating category. At the same time, we consider the history of news releases for each firm.

Enron Corporation provides products and services related to natural gas, electricity and communications. For most of late 2001, Enron was front-page news for its spectacular fall. In a period of less than a year, Enron stock fell from USD 83.13 to less than a dollar. It was the largest firm to file for bankruptcy protection in U.S. history.

Figure 4.3 shows both Enron's CreditGrades DP and its rating history. During most of 2000, Enron's DP was stable, averaging 1.47 percent while during that same period the median DP for A-rated companies was 3.75 percent. Enron stock reached its highest level ever with USD 90.00 on August 23, 2000.

In early 2001, as Enron's attempts to extend its lines of business became unprofitable, its stock price fell. In addition, California was in the midst of a severe energy crisis and as more power plants were being built to meet the demand, Enron's stock further slipped on fears that an energy glut would cut profits. Because of these concerns, its CreditGrades DP began to slowly increase due to changing market sentiments. By August 14, when Enron President and CEO Jeffrey Skilling resigned after just six months on the job, the DP had risen to nearly 9.00 percent. Enron stock had fallen to USD 42.92.

In the following months, the market received a series of signals on the changing credit quality of Enron. On October 16, Enron reported a third quarter loss, its first quarterly loss in over four years, after taking charges of USD 1 billion on poorly performing businesses. This was followed by an announcement by the SEC on October 22 that it would begin looking into Enron's off-balance sheet partnerships. On October 29, 2001, employee retirement plans were frozen. These resulted in further lowering market confidence in Enron's

Figure 4.3: Enron. CreditGrades™ and S&P implied 5-year probability of default.

ability to meet its debt obligations. After starting October at 15 percent, the Enron CreditGrades DP had finished the month at 31 percent.

On November 8, Enron announced that it would restate earnings to a USD 600 million loss for the period 1997–2001. The market had already priced this and the stock price fell only to USD 8.41 from USD 9.05 the previous day. The DP stayed in the mid-30 percent range. However, in its filing of its 10-Q with the SEC on November 19, Enron reduced previously reported third quarter 2001 earnings by USD 0.03 per diluted share, causing its stock price to tumble and its DP to climb to 45 percent. Within a few days, on November 28, Dynegy Inc. announced that it would terminate its previously announced merger agreement with Enron, citing Enron's breaches of representations, warranties, covenants and agreements in the merger agreement. The DP spiked to over 75 percent over a two-day period.

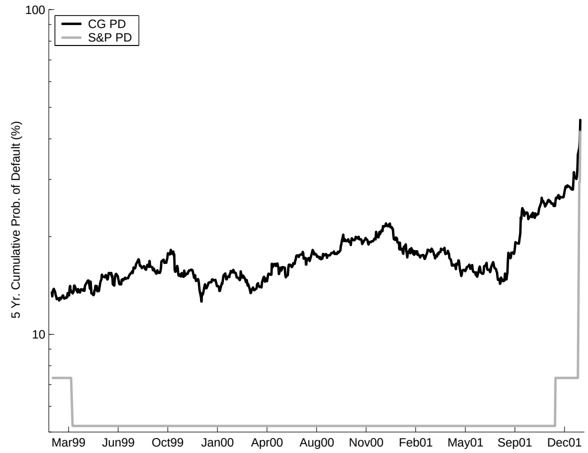
On December 2, 2001, Enron Corp. filed for Chapter 11 protection. Its DP was now over 70 percent.

4.4 Kmart

Kmart Corporation is third-largest U.S. discount retailer behind Wal-Mart Stores Inc., which leads the basic discounting business, and Target Corp., which lures a slightly more affluent customer. The company operates in the general merchandise retailing industry through 2,105 discount stores. Its bankruptcy in 2002 was the largest retail bankruptcy proceeding on record.

In recent years Kmart has struggled with declining sales, outdated technology, and a heavy debt burden. During the mid 1990s, Kmart avoided bankrupty by selling off Office Max, the Sports Authority and other retailers acquired in the ‘80s and early ‘90s. During this period, its debt was rated BB by S&P.

Figure 4.4: Kmart. CreditGrades™ and S&P implied 5-year probability of default.



Since the beginning of 1999, Kmart's 5-year DP had drifted between 13 percent and 22 percent as it struggled to compete with its rivals and meet its debt obligations (see Figure 4.4). For two consecutive years, annual net income had declined, and three consecutive quarters of negative income in 2001 helped push Kmart into bankruptcy.

On August 23, the quarterly report for the period ending August 1 was released, reporting a net loss of USD 95 million. The DP increased from 17 to 24 percent over the next few weeks amid investor concerns that the company would not be able to meet its short-term obligations in the recessionary environment. This was the firm's steepest increase for a one-month period within the past two years, signaling a significant shift in the market's perception of Kmart's credit quality. In contrast, S&P did not downgrade Kmart until November 27, over two months later.

On November 27, Kmart filed its quarterly report for the period ending October 31. A net loss of USD 224 million was reported. The company had USD 366 million in cash on its balance sheet but owed its suppliers USD 3.3 billion.

In January, Kmart reported same-store sales were down 1 percent from the previous year during December, the crucial holiday-shopping month. In contrast, Wal-Mart Stores Inc. reported that December same-store sales rose 8 percent and Target Corp.'s discount unit posted a 10 percent jump in sales. By January 3, 2002, the Kmart DP edged above 30 percent as investors worried that with dropping sales, Kmart would not be able to sell enough inventory to pay off its suppliers. S&P downgraded Kmart from BB to B- approximately two weeks later. Within a week, Kmart filed for Chapter 11 bankruptcy protection.

4.5 Rite Aid

Rite Aid Corporation is the third-largest retail drugstore chain in the United States based on sales, behind CVS Corp. and Walgreen Co. From the beginning of fiscal 1997 until December 1999, Rite Aid pursued an aggressive expansion plan, including the purchase of PCS Health Systems in November 1998 for USD 1.5 billion, placing itself in heavy debt. Debt (short-term and long-term) increased from USD 2.4 billion in May 1997 to USD 3.4 billion by the end of 1998. During 1999, a series of lawsuits and negative earnings surprises eroded Rite Aid management's credibility with the market.

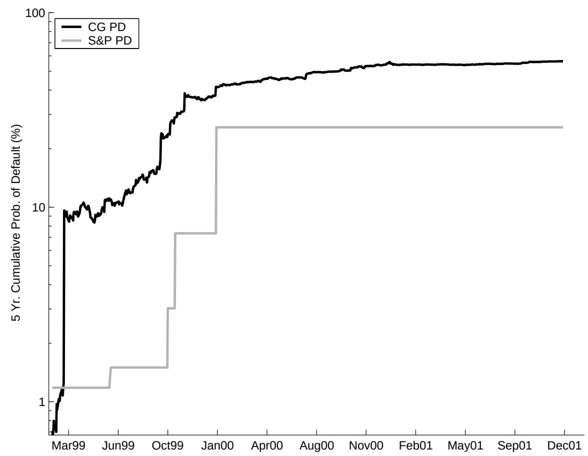
On March 12, 1999, Rite Aid announced earnings for the 4th quarter ending February 2, 1999, would be between USD 0.30 and USD 0.32 per share, well below analysts' expectations of USD 0.52. In a one day period, Rite Aid stock fell USD 14.44 (39 percent). Within a few days, several class action lawsuits were filed against the company, alleging that Rite Aid issued false and misleading statements concerning operating results and business prospects. The DP jumped from 1.25 percent to 9.63 percent within a day as the announcement raised questions regarding the company's ability to control its fast-growing operations. Three months after the initial signal from the market, on June 14, S&P downgraded Rite Aid from BBB+ to BBB.

For the next several months, the CreditGrades DP climbed amid disclosures that USD 35 million (42 percent) of pretax profit for the quarter ending February 28 did not come from operations but from one-time gains related to a settlement of a lawsuit with suppliers. This was followed by news that a class action lawsuit was filed against Rite Aid accusing the company of selling outdated merchandise. On September 22, another suit was filed, this time by the Florida attorney general's office, alleging theft, wire fraud, unfair and deceptive trade practices, and racketeering by overcharging 29,000 uninsured prescription drug customers. Following this news, the DP jumped to 24 percent, marking yet another significant change in the market's sentiments on Rite Aid's credit quality. Two weeks later, on October 7, S&P downgraded the company to BBB-.

On October 12, Rite Aid restated its earnings for the second quarter of 1999, stating a loss of USD 0.26 per share; analysts had predicted USD 0.24 per share. The company also stated that it would again restate earnings for the past three years, reducing pretax profit by USD 500 million. The CreditGrades DP increased 4 percent to 27 percent over the period of three days, another strong signal of declining market confidence in Rite Aid. Within a few days, on October 18, CEO Martin Grass resigned. On October 22, S&P downgraded Rite Aid to BB.

On November 10, Rite Aid once again stunned the market by abruptly canceling a conference call with analysts while stating that investors should not rely on the "forward looking profitability and cashflow information" presented in the prior month, further eroding its credibility with the market and sending the DP soaring to 39 percent. Shortly after, KPMG, Rite Aid's auditing firm, resigned. On January 11, 2001, Rite Aid announced it would delay issuing financial statements until as late as the summer. S&P reacted to this latest news by downgrading Rite Aid to B on January 13.

Figure 4.5: Rite Aid Corp. CreditGrades™ and S&P implied five-year probability of default.



4.6 Lucent Technologies

Lucent Technologies Inc. is one of the world's largest telecommunications equipment companies. Spun off from AT&T Corp. in 1995, Lucent had been a blue-chip stock on Wall Street with a stock price that seemed to double every year. Beginning in 2000, the company began to experience a series of problems trying to adjust to newer technology and weakening client demand. In that year, Lucent's stock steadily plunged, effectively eliminating the company's ability to raise more cash with stock sales to ease its debt load.

Throughout 2000, Lucent had been a very volatile stock due to increasing investor concerns over earnings. In January, the markets had already punished Lucent for missing its earnings target for the first quarter by sending the stock price down 31 percent over a four-day period. In mid-July, Lucent warned that profit and revenue growth for the rest of the year would be slower than had been expected. Lucent's stock price fell from USD 64.69 on July 19 to USD 40.50 by August 14. The CreditGrades 5-year DP increased from 0.22 percent to 0.90 percent, marking a significant change in credit sentiment. Up to this point, Lucent's DP had been historically around 0.20 percent. At the time, S&P rated Lucent bonds single-A and would not downgrade the bonds until later in December after the market sent out several other signals of changing market sentiment.

During the period from July to October of 2000, several telecommunications analysts reported signs that the sector may be poised for a slowdown. The major phone companies were seeing a slowdown in revenue and, as a result, were planning to cut equipment purchases. For example, revenue at established carriers such as AT&T and WorldCom grew slower than was expected earlier in the year, largely because of intense price competition for consumers' long-distance business. Meanwhile, some new entrants into the market were having trouble raising capital to build their networks and compete with the larger carriers. Also, as the telecommunications sector consolidated, the newly merged giants were able to use their size to demand ever steeper discounts. Compounding the problem, some of Lucent's customers were delaying payment for the equipment they purchased and the company was slow to collect on its bills, suppressing the amount of cash it had on hand.

Lucent acknowledged that it had missed the opportunity to develop the technology clients wanted and was now trying to catch up with competitors such as Nortel. In an attempt to reorganize, Lucent announced that it would spin off several divisions in an effort to refocus its core businesses as customer demand switched from one technology to another. In addition, it was believed that by spinning off divisions such as the microelectronics unit, which made 25 percent of its sales to other Lucent divisions, Lucent would be free to purchase its components from independent suppliers who might have newer or cheaper products. Despite its efforts, over this period, the CreditGrades DP slowly increased to 2 percent.

In October, Lucent surprised the market again by posting earnings of USD 0.17 per share instead of the USD 0.32 that analysts had expected. Lucent also admitted that it had increased its reserves for bad debt as start-up phone companies continued to fail, once again raising investor concerns over vendor financing.[3] The news resulted in the CreditGrades DP surging to 5.78 percent.

For the remainder of 2000, Lucent's recovery was hampered by the continued weakening of its traditional markets, high employee turnover and uncertainty about whether executives could carry out the radical transformation that the company needed. In addition, analysts were raising concerns over a buildup in inventory, the looming obsolescence of key products and customers taking longer to pay bills.

On November 20, Lucent revealed that it had overstated fourth-quarter profits due to prematurely recorded USD 125 million worth of sales. Lucent also erased USD 452 million in sales of equipment returned by distributors unable to sell it. Further bad news followed in the December annual report when Lucent stated that for fiscal 2000, USD 892 million was set aside for gear it could not sell, compared with USD 709 million at the end of 1999. It also saw its average wait for payment expand to 102 days from 89 days.

On December 21, S&P downgraded Lucent to BBB+, five months after the first strong indications of changing market sentiment. The CreditGrades DP had increased to over 12.00 percent.

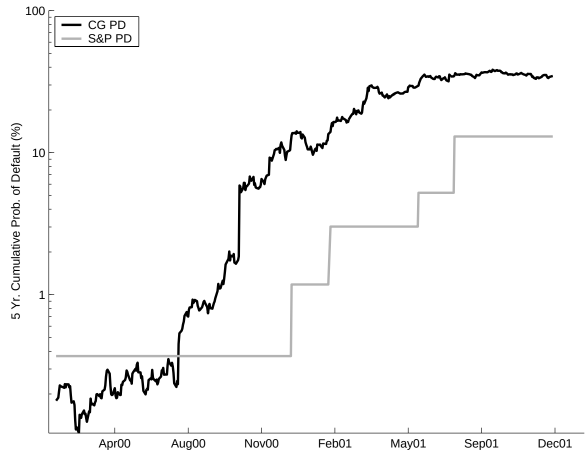
Problems continued to mount for Lucent in 2001. At the beginning of 2001, the Securities and Exchange Commission started probing into possible fraudulent accounting practices in booking sales and on February 12, 2001, S&P downgraded Lucent to BBB-.

In March 2001, Lucent spun off Agere Systems Inc. in an attempt to raise cash to pay down its debt. However, given the weak capital markets, the IPO raised only USD 6 per share, far less than the original goal of USD 15 to USD 20 Lucent had sought. Within days, S&P announced that it would place Lucent under review for a possible cut to junk status. During this period, several sources reported that Lucent had begun drawing on its bank credit lines, a move that suggested that it may have used much of the USD 3.8 billion of cash it had at the end of 2000. The CreditGrades DP climbed from 20 percent to 30 percent within two weeks.

[3] At the end of December, Lucent disclosed that it had increased its provision for questionable debts to USD 252 million for fiscal year ended September 30 from USD 67 million the previous year.

On June 12, 2001, S&P downgraded the company to BB+. S&P cited deteriorating conditions in the telecommunications equipment sector in the U.S. and in Europe.

Figure 4.6: Lucent. CreditGrades™ and S&P implied 5-year probability of default.



Appendix A

Calculation of Credit Default Swap Spreads

In this appendix we derive the closed-form price formula for a CDS within the context of the closed-form asset based model.

From (2.28) we have that the survival probability up to time t is

$$P(t) = \Phi\left(-\frac{A_t}{2} + \frac{\log(d)}{A_t}\right) - d \cdot \Phi\left(-\frac{A_t}{2} - \frac{\log(d)}{A_t}\right), \tag{A.1}$$

where

$$d = \frac{S_0 + \bar{L}D}{\bar{L}D} e^{\lambda^2}, \tag{A.2}$$

$$A_t^2 = \left(\sigma_S^* \frac{S^*}{S^* + \bar{L}D}\right)^2 t + \lambda^2. \tag{A.3}$$

The probability density function for the default time is defined as:

$$f(t) = -\frac{dP(t)}{dt}, \tag{A.4}$$

so that the cumulative default probability up to time t is given by

$$1 - P(0) + \int_0^t ds f(s). \tag{A.5}$$

Consider a CDS with maturity t and a continuous spread c. The discounted expected loss payments for the CDS are given by

$$(1 - R)\left[1 - P(0) + \int_0^t ds f(s) e^{-rs}\right], \tag{A.6}$$

where R is the asset specific recovery rate and r is the default-free discount rate.

The discounted expected spread payments are given by

$$c \int_0^t ds P(s) e^{-rs}. \tag{A.7}$$

The price of the CDS is then given by the difference of the discounted spread and loss payments:

$$CDS = (1-R)\left[1 - P(0) + \int_0^t ds f(s) e^{-rs}\right] - c\int_0^t ds e^{-rs} P(s). \tag{A.8}$$

We can write (A.8) as

$$CDS = (1-R)(1-P(0)) - \frac{c}{r}\left(P(0) - P(t)e^{-rt}\right) + \left(1 - R + \frac{c}{r}\right)\int_0^t ds e^{-rs} f(s). \tag{A.9}$$

It turns out that there is a closed form expression for the integral in (A.9),

$$\int_0^t ds e^{-rs} f(s) = e^{r\lambda^2/\sigma^2}\left[G(t + \lambda^2/\sigma^2) - G(\lambda^2/\sigma^2)\right]. \tag{A.10}$$

The function $G(t)$ is given by[1]

$$G(t) = d^{z+1/2}\Phi\left(-\frac{\log(d)}{\sigma\sqrt{t}} - z\sigma\sqrt{t}\right) + d^{-z+1/2}\Phi\left(-\frac{\log(d)}{\sigma\sqrt{t}} + z\sigma\sqrt{t}\right), \tag{A.11}$$

where $z = \sqrt{1/4 + 2r/\sigma^2}$. Combining (A.9) and (A.10), we have

$$CDS = (1-R)(1-P(0)) - \frac{c}{r}\left(P(0) - P(t)e^{-rt}\right) + \left(1 - R + \frac{c}{r}\right) e^{r\xi}(G(t+\xi) - G(\xi)), \tag{A.12}$$

with $\xi = \lambda^2/\sigma^2$. We find the par spread c^* that makes the CDS price equal to zero:

$$c^* = r(1-R)\frac{1 - P(0) + H(t)}{P(0) - P(t)e^{-rt} - H(t)} \tag{A.13}$$

where

$$H(t) = e^{r\xi}(G(t+\xi) - G(\xi)). \tag{A.14}$$

[1] See Rubinstein and Reiner (1991).

Appendix B

Calculation of Debt-per-Share

This appendix describes the algorithm for the debt-per-share calculation that is used for industrial companies. The debt-per-share is defined as the liabilities that participate in the financial leverage of the firm divided by the equivalent number of outstanding shares. It is determined based on financial data from consolidated statements. Using Bloomberg data, the financial debt is estimated by

$$\begin{aligned} \text{Financial\_Debt} &= \text{ST\_Borrow} + \text{LT\_Borrow} + 0.5 * (\text{Other\_ST\_Liabilities} + \text{Other\_LT\_Liabilities}) \\ &\quad + 0 * (\text{Acct\_Payable}). \end{aligned} \tag{B.1}$$

ST_Borrow and LT_Borrow are the short-term and long-term interest-bearing financial obligations including bank overdrafts, bonds, loans, etc. Other_ST_Liabilities and Other_LT_Liabilities represent current and long-term obligations that do not bear explicit interest, such as tax liabilities and pension liabilities. We use a 50 percent weight for Other_ST_Liabilities and Other_LT_Liabilities, as some of these are similar to financial liabilities (pension liabilities, leases, etc.) while some of them are not (deferred taxes, provisions, etc). We use a 0 percent weight for accounts payable as they do not participate in the financial leverage of a company.

The liabilities of subsidiaries are consolidated at 100 percent on a consolidated balance sheet even though the parent company may not own 100 percent of the subsidiary. To adjust for this, we assume that the subsidiary has a debt-to-equity ratio of k. Thus,

$$\text{Minority\_Debt} = k * \text{Minority\_Interest}, \tag{B.2}$$

where Minority_Interest represents the portion of interest the parent company does not own in the subsidiary. The total debt used in the DPS calculation is then given by

$$\text{Debt} = \text{Financial\_Debt} - k * \text{Minority\_Interest}. \tag{B.3}$$

In the calculation, we assume $k = 1$ and limit Minority_Debt to no more than half of Financial_Debt.

The total number of shares is the sum of all classes common shares and the equivalent number of common shares to account for the preferred equity, namely,

$$\text{Number\_of\_Shares} = \text{Common\_Shares} + \text{PFD\_Shares}. \tag{B.4}$$

Common_Shares is given by the current market cap divided by the current stock price,

$$\text{Common\_Shares} = \text{Market\_Cap}/\text{Stock\_Price}. \tag{B.5}$$

In Bloomberg, the current market cap includes the market value of all classes of common equity. The equivalent number of preferred shares is given by

$$\text{PFD\_Shares} = \text{PFD\_Equity}/\text{Stock\_Price}, \tag{B.6}$$

where PFD_Equity is the book value of the preferred shares and stock price reference is the price of the common stock when the preferred book value was calculated. For simplicity, one can use the current stock price as the reference as long as it has not significantly changed since the last financial statement. In the calculation, PFD_Shares is capped at half of Common_Shares. Finally, debt-per-share is calculated from (B.3) and (B.4) by

$$\text{Debt\_per\_Share} = \text{Debt}/\text{Number\_of\_Shares}. \tag{B.7}$$

The debt-per-share calculation is thus quite simple requires only the following fields from Bloomberg:

- BS_ST_BORROW
- BS_LT_BORROW
- BS_OTHER_ST_LIAB
- BS_OTHER_LT_LIAB
- BS_MINORITY_INT
- CUR_MKT_CAP
- BS_PFD_EQY
- PX_LAST

This algorithm is used for industrial firms and is not necessarily valid for financial firms. For certain specific companies, which have an important financial subsidiary (for example, auto manufacturers), we use a debt-per-share override that excludes the financial subsidiary from the consolidated debt.

Appendix C

Sample CreditGrades™ Grid

The table below displays CreditGrades spreads for a variety of stock-to-debt (S_0/D) ratios and equity volatility levels. In all cases, we have used the standard CreditGrades assumptions of $\bar{L} = 0.5$, $\lambda = 0.3$, and $R = 0.5$, and have assumed an interest rate of 5 percent.

Table C.1: CreditGrades™ Table (basis points)

	Equity volatility (percent)												
S_0/D	20	25	30	35	40	45	50	55	60	65	70	75	80
0.5	55	85	125	175	232	297	367	441	520	602	687	774	865
1.0	8	22	46	82	130	188	253	326	403	486	572	662	755
1.5	2	8	22	48	85	134	193	260	333	412	495	583	675
2.0	1	3	12	30	59	101	153	214	283	358	438	523	612
2.5	0	2	7	20	43	78	124	180	244	315	392	474	561
3.0	0	1	4	13	32	62	103	154	214	282	355	434	518
3.5	0	0	3	9	24	50	86	133	190	254	325	401	483
4.0	0	0	2	7	19	41	73	117	169	230	298	373	452
4.5	0	0	1	5	15	34	63	103	152	211	276	348	425
5.0	0	0	1	4	12	28	55	91	138	194	257	326	401
5.5	0	0	1	3	10	24	48	82	126	179	240	307	381
6.0	0	0	0	2	8	20	42	74	115	166	224	290	362

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